8. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, The Motion of Liquids and Gases in Natural Strata [in Russian], Nedra, Moscow (1984).
9. H. L. Beckers and G. J. Harmsen, "The effect of water injection on sustained combustion in a porous medium," Soc. Petrol. Eng. J., 10, No. 2 (1970).
10. O. M. Alishaeva, V. M. Entov, and A. F. Zazovskii, "The structure in conjugate saturation and concentration steps in oil displacement by active solutions," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1982).
11. P. G. Bedrikovetskii and M. V. Lur'e, "Discontinuity stability and permissibility in equation systems for two-phase infiltration," Prikl. Mat. Mekh., 47, No. 4 (1983).
12. I. M. Gel'fand, "Some problems in quasilinear-equation theory," Usp. Mat. Nauk, 14, No. 2 (1959).

INCREASE IN WATER-HAMMER PRESSURE IN A PIPE IN THE PRESENCE OF A LOCALIZED VOLUME OF GAS

S. P. Aktershev and A. V. Fedorov

UDC 532.595.2+532.595.7

In different areas of use of piping systems, situations are often encountered whereby localized volumes of gas co-exist with the liquid in the pipe. The presence of the gas cavities may have a significant effect on the character of various transients in the pipeline [1-8]. Gas cavities may compensate for pressure fluctuations [2] or, conversely, may increase the maximum pressure in the pipe [3, 4]. The exact role played by the cavities depends on the parameters of the system and the method of organization of the nonsteady flow. As is known [1], the air chamber installed in the delivery line immediately after a pump reduces the pressure jump which occurs when the pump is started. On the other hand, when a capped pipe is filled with liquid, the presence of gas may lead to a water hammer of considerable magnitude [3]. The presence of air at the end of a delivery line with a closed valve may also result in large pressure fluctuations when the pump is quickly turned on [4].

The pressure-testing of a pipeline filled with a viscous liquid and provided with an air chamber (Fig. 1) was studied experimentally in [5] for large values of friction at the point of attachment of the chamber to the line. Valve A, connecting the line, under the pressure $\tilde{p}_0$, with a tank under constant pressure $\tilde{p}_1 > \tilde{p}_0$, was quickly opened at the initial moment of time. The air chamber was designed to damp the attendant pressure oscillations. The experimental data was compared with the results of numerical calculations. It was found that the maximum pressures were 1.5-1.8 times higher within a certain range of volumes of air in the chamber than in the absence of air. The results of the numerical calculations were used to determine the maximum permissible diameter of chamber throat that would ensure damping of pressure oscillations by the chamber for a specified volume of air.

Here we also examine the problem of the pressure-testing of a pipeline with a gas cavity. However, we will use small values of friction and assume that friction is concentrated in the initial section of the pipe (valve resistance). The effect of the volume of the gas cavity on the maximum pressures in the pipeline is studied both by a numerical method and within the framework of a simplified mathematical model proposed below.

Formulation of the Problem. The flow of liquid in the pipe is described by hydraulic equations [1] which appear as follows in the dimensionless variables $p = \tilde{p}/\tilde{p}_1$, $u = \tilde{\rho}_0\tilde{c}\tilde{u}/\tilde{p}_1$, $x = \tilde{x}/\tilde{L}$, $t = \tilde{c}\tilde{t}/\tilde{L}$

$$\frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} + \alpha u|u| = 0, \quad \alpha = \lambda\tilde{L}\tilde{p}_1/(2\tilde{D}\tilde{\rho}_0\tilde{c}^2). \tag{1}$$

Here, $\tilde{p}$, $\tilde{u}$, $\tilde{\rho}_0$, $\tilde{x}$, $\tilde{t}$ are the dimensionless pressure, velocity, and density of the liquid, the longitudinal coordinate, and time; $\tilde{D}$, $\tilde{L}$, $\tilde{c}$ are the diameter and length of the pipe and the rate of propagation of perturbations in the pipe when it is filled with liquid; λ is the coefficient of friction against the wall. We assume that λ is constant, which is valid for Reynolds numbers $\mathrm{Re} \geqslant 10^5$ [9].

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 107-111, November-December, 1987. Original article submitted June 16, 1986.

0021-8944/87/2806-0899$12.50

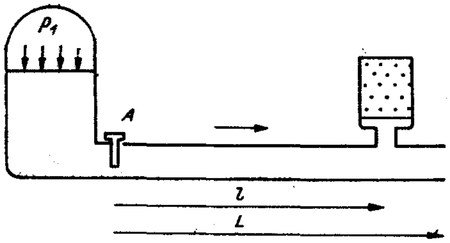

Fig. 1

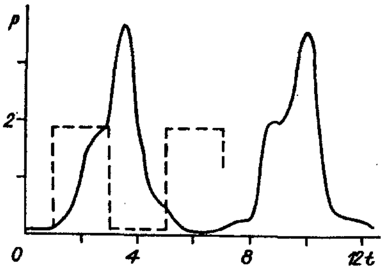

Fig. 2

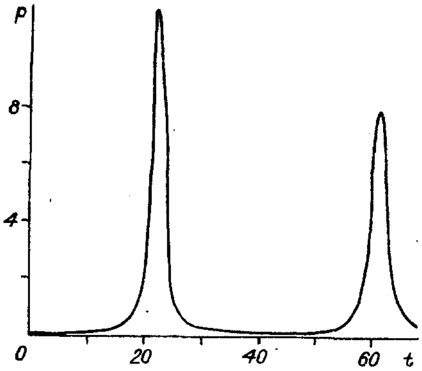

Fig. 3

In the section in which the gas cavity is located, we write the equation describing the change in the volume of gas

$$d\tilde{V}/d\tilde{t} = -\tilde{f}_0(\tilde{u}^- - \tilde{u}^+),$$

where $\tilde{V}$ is the gas volume; $\tilde{f}_0$ is the cross-sectional area of the pipe; $\tilde{u}^-$, $\tilde{u}^+$ are the velocities of the liquid to the left and right of the cavity. The compression of the gas volume is assumed to be adiabatic $\tilde{p}\tilde{V}^\gamma = \tilde{p}_0\tilde{V}_0^\gamma$. From here, we obtain a relation for $\tilde{u}^-$, $\tilde{u}^+$, $d\tilde{p}/d\tilde{t}$ which takes the following form in dimensionless variables

$$dp/dt = \varkappa p^{(1+\gamma)/\gamma}(u^- - u^+), \quad \varkappa = \frac{\gamma \tilde{f}_0 \tilde{L} \tilde{p}_1}{\tilde{V}_0 \tilde{\rho}_0 \tilde{c}^2}\left(\frac{\tilde{p}_1}{\tilde{p}_0}\right)^{1/\gamma}. \tag{2}$$

In the initial section of the pipe, the dimensionless pressure and velocity are connected by the relation

$$1 - p = \xi(\tilde{p}_1/2\tilde{\rho}_0\tilde{c}^2)u|u|. \tag{3}$$

Here, ξ (the friction coefficient of the valve [19]) is also assumed to be constant.

In the final section of the pipe

$$u(1, t) = 0. \tag{4}$$

The initial conditions in the pipe:

$$u(x, 0) = 0,\ p(x, 0) = p_0,\ p_0 = \tilde{p}_0/\tilde{p}_1. \tag{5}$$

We used the method of characteristics [10] to solve (1) with boundary and initial conditions (3)-(5). Here, conditions (2)-(4) were augmented by relations for the characteristics in the corresponding sections.

Some Results of the Calculations. Calculations were performed for a pipe filled with water ($\tilde{C}$ = 1370 m/sec, $\tilde{\rho}_0 = 10^3$ kg/m^3) under a pressure $\tilde{p}_0$ = 0.1 MPa and having an air chamber (γ = 1.4) at its end ($\tilde{l}/\tilde{L}$ = 0.96), with $\tilde{L}/\tilde{D}$ = 100, λ = 0.02, ξ = 0.4; 1, 2.

Figure 2 shows the dependence of the pressure in the gas cavity on time for p_0 = 0.1, $\tilde{V}_0/\tilde{f}_0\tilde{L} = 10^{-3}$, ξ = 1. As is known [1], in the absence of gas, the maximum inertial increase in pressure in the pipe will be equal to $2(1 - p_0)$ if we ignore the losses. The period of the oscillations will be four wavelengths, as illustrated by the dashed line in Fig. 2. It is evident that in the presence of a gas cavity, pressure peaks of great amplitude are realized.

Here, the pressure in the first peak p_m and the period of the oscillations (the interval between successive peaks) depends appreciably on the initial volume of the gas. Figure 3 shows the results of calculations for p = 0.1, $\tilde{V}_0/\tilde{f}_0\tilde{L} = 10^{-1}$, ξ = 2. The pressure in the first peak is more than 10 times the pressure in the tank. The pressure reduction in subsequent pulses is determined by the friction losses ξ and $\lambda\tilde{L}/\tilde{D}$.

The same pressure oscillations take place in the pipe sections next to the gas cavity (the maximum pressure is reached in the end section). This phenomenon of the amplification of water-hammer pressure due to the presence of a gas volume in a pipe can be used in equipment to develop impulsive jets (in the cutting of metal by a fluid jet, in monitors for excavating mineral deposits, etc.) and to test containers under high-pressure [11, 12].

Model of a Rigid Piston. We will examine a simplified mathematical model to analytically describe the problem being studied here.

The liquid in the pipe is assumed to be incompressible and is regarded as a rigid piston. Here, the velocity of the liquid is the same in all sections of the pipe. Thus, we ignore the time of propagation of disturbances over the pipeline. Ignoring the compressibility of the liquid and the compliance of the pipe wall compared to the compressibility of the gas is valid under the condition that the characteristic time of the process is much greater than the time it takes for a wave to travel over the pipeline.

Let the gas cavity be located at the end of the pipe ($\tilde{l} = \tilde{L}$), $\tilde{u}$ is the velocity of the column of liquid and $\tilde{p}$ is the pressure in the gas cavity (see Fig. 1). With allowance for the pressure loss at the pipe inlet and friction against the wall, we write the equation of motion of the liquid piston as follows

$$\tilde{\rho}_0 \tilde{L} \frac{d\tilde{u}}{d\tilde{t}} = \tilde{p}_1 - \tilde{p} - \left(\xi + \frac{\lambda \tilde{L}}{\tilde{D}}\right) \frac{\tilde{\rho}_0 \tilde{u} |\tilde{u}|}{2},$$

while the change in the volume of the gas

$$d\tilde{V}/d\tilde{t} = -\tilde{f}_0 \tilde{u},$$

and the adiabatic compression of the gas

$$\tilde{p}\tilde{V}^{\gamma} = \tilde{p}_0 \tilde{V}_0^{\gamma}.$$

Excluding the volume $\tilde{V}$, we obtain two equations for $\tilde{p}$ and $\tilde{u}$. Ignoring the dimensionless variables, we can write these equations in the form

$$du/dt = 1 - p - ku|u|, \quad dp/dt = \varkappa u p^{(1+\gamma)/\gamma}, \tag{6}$$

where $k = \left(\xi + \frac{\lambda \tilde{L}}{\tilde{D}}\right) \frac{\tilde{p}_1}{2\tilde{\rho}_0 \tilde{c}^2}$; $\varkappa = \frac{\gamma \tilde{f}_0 \tilde{L} \tilde{p}_1}{\tilde{V}_0 \tilde{\rho}_0 \tilde{c}^2} \left(\frac{\tilde{p}_1}{\tilde{p}_0}\right)^{1/\gamma}$. These equations can be integrated by numerical methods with the initial data $p(0) = p_0$, $u(0) = 0$. We will analyze (6) for the case of the absence of loss ($k = 0$) and we will linearize (6) near the stationary point $p = 1$, $u = 0$, having put $p = 1 + p'$, $u = u'$ (p' and u' are small quantities). For p' we obtain the equation of harmonic vibrations with the period

$$T = 2\pi/\sqrt{\varkappa}. \tag{7}$$

This result coincides with the result obtained in [8] for small oscillations in a pipeline with a gas compensator.

The oscillatory character of the process is due to the inertia of the liquid and the presence of the gas cavity. After the valve is opened, the pressure in the tank forces the column of liquid to the right (see Fig. 1) and compresses the gas, which acts as an elastic spring. Due to the inertia of the liquid, the equilibrium position "jumps" (when the gas pressure is comparable to the pressure at the pipe inlet), and the gas is compressed to $p_m > 1$ by the time the liquid decelerates completely. In the absence of a gas cavity, the entire liquid acquires the velocity $(1 - p_0)$ during one passage of the wave over the length of the pipeline. The column is then slowed by the wave as it is reflected from the closed end of the the line [1]. In the case being studied here (presence of gas), the liquid is accelerated for a much longer period of time and thus acquires a much greater velocity during acceleration. Consequently, when it is slowed, the pressure created is greater by a factor of $2(1 - p_0)$ than the pressure attained in the absence of the gas.

We obtain the pressure p_m in the first peak from (6) for $k \neq 0$, having examined the motion of the liquid at $u \geqslant 0$. Introducing the new variables z = (1/,* $y = u^2/2$ and excluding the time t, we have

$$\frac{dy}{dz} - \beta y = \frac{\gamma}{\varkappa}\left(\frac{1}{z^{\gamma}} - 1\right), \quad \beta = \frac{2k\gamma}{\varkappa}. \tag{8}$$

We write the solution of (8), with the initial conditions $y(z_0) = 0$ ($z_0 = (1/p_0)^{1/\gamma}$), in the form

*Expression missing in Russian original — Publisher.

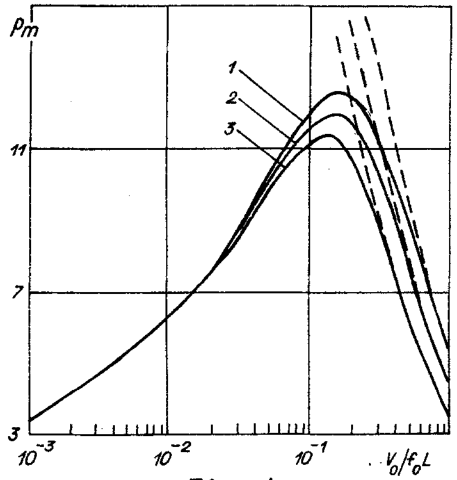

Fig. 4

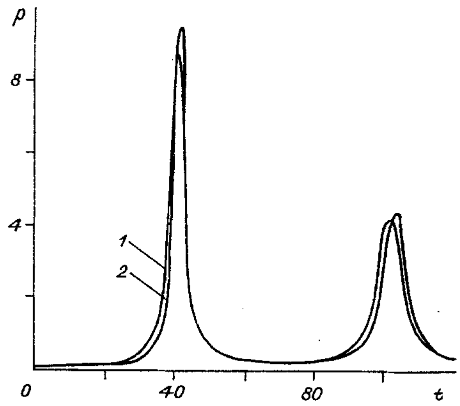

Fig. 5

$$y(z)=\frac{\gamma}{\varkappa}\,e^{\beta z}\int\limits_{z}^{z_0}\left(1-\frac{1}{\eta^{\gamma}}\right)e^{-\beta\eta}\,d\eta,\quad z<z_0. \tag{9}$$

It is not hard to show that the function y(z) vanishes at the point $z_m < 1$, $y(z_m) = 0$, is nonnegative on the interval (z_m, z_0), and has its maximum in this interval. Having determined z_m, we find $p_m = 1/z_m^{\gamma}$.

Hydraulic losses of pressure at pipe inlet and friction losses on the wall, accounted for in the first equation of (6) by the coefficient k, compensate partially for the inertia of liquid piston and thereby reduce the magnitude of the water hammer. Let us evaluate the time for piston velocity to relax to the steady-state value as a result of the losses, assuming in (6) that $u \geqslant 0$ and that the gas pressure is constant $[p(t) = p_0]$.

The solution of the equation $du/dt = 1 - p_0 - ku^2$, with the initial condition $u(0) = 0$, will be $u(t)=u^*\left(\frac{e^{t/\tau}-1}{e^{t/\tau}+1}\right)$. Here, $\tau = 1/[2\sqrt{k(1-p_0)}]$ is the relaxation time; $u^* = \sqrt{(1-p_0)/k}$ is the steady-state velocity at which the inertial term vanishes. If the characteristic time of the change in gas pressure $T \ll \tau$, then there will not be sufficient time for the velocity of the liquid to "adjust" to the current value of pressure. The value of τ/T decreases with an increase in the volume of the gas $\tilde{V}_0$. Thus, friction losses play a more substantial role for large relative volumes $\tilde{V}_0/\tilde{f}_0\tilde{L}$ in the liquid piston model.

Comparison of Results Calculated by the Method of Characteristics and from the Piston Model. Figure 4 shows the relations $p_m(\tilde{V}_0/\tilde{f}_0\tilde{L})$ for $p_0 = 0.1$ calculated by the method of characteristics (solid lines 1-3 for $\xi = 0.4$, 1, and 2) in comparison with results calculated using the piston model (9) (dashed lines). It is evident that with an increase in gas volume the value of p_m calculated by the characteristics method initially increases (the effect of air on the water hammer increases) and then decreases due to intensification of the effect of friction losses. Thus the most critical volume of gas $\tilde{V}_0^*$ for the water hammer (and optimal for obtaining impulse pressure) is determined by friction losses. In the case of large volumes the air chamber damps pressure oscillations.

With small volumes ($\tilde{V}_0/\tilde{f}_0\tilde{L} < 10^{-2}$), the same value of p_m is obtained for different ξ because the characteristic time of pressure build-up is fairly small and the effect of friction losses is insubstantial. At $\tilde{V}_0/\tilde{f}_0\tilde{L} > 0.8$, the piston model agrees adequately with the characteristics method in terms of p_m. It can be seen from Fig. 4 that the piston model is invalid at $\tilde{V}_0 \ll \tilde{V}_0^*$, when the characteristic time of the process becomes comparable to the time required for the wave to travel the length of the pipeline.

Figure 5 shows the dependence of the pressure in the gas cavity on time, calculated by the characteristics method and from Eq. (6) (curves 1 and 2) for $p_0 = 0.1$, $\tilde{V}_0/\tilde{f}_0\tilde{L} = 0.3$, $\xi = 2$.

Thus, we studied the process of pressure oscillation in a liquid-filled pipe containing a gas cavity in the case of small values of system friction. We determined the critical volume of gas at which the increase in pressure associated with water hammer is greatest.

LITERATURE CITED

1. I. A. Charnyi, Unsteady Motion of a Real Liquid in Pipes [in Russian], Nauka, Moscow (1975).
2. Kh. N. Nizamov, G. L. Batin, et al., "Promising methods of damping pressure oscillations in pipes," in: Hydroelastic Vibrations and Methods of Eliminating Them in Closed Piping Systems [in Russian], Krasnoyarsk (1983).
3. V. V. Berdnikov, T. S. Kozyreva, and B. A. Pantyukhin, "Study of the filling of a main line with liquid," Izv. Vyssh. Uchebn. Zaved. Aviats. Tekh., No. 3 (1982).
4. T. Tomita and K. Nakamura, "Water hammer caused by air in a pump started with the discharge valve closed," Ebara Eng. Rev., No. 127 (1984).
5. A. Kitagawa, "A method of absorption for surge pressure in conduits," Bull. JSME, 22, No. 165 (1979).
6. S. P. Aktershev, A. V. Fedorov, and V. M. Fomin, "Mathematical modeling of tests of main pipelines for air-tightness with allowance for trapped volumes of air," in: Dynamics of Multiphase Media [in Russian], Novosibirsk (1985).
7. S. P. Aktershev, A. V. Fedorov, and V. M. Fomin, "Mathematical modeling of the filling of a main pipeline," in: Summary of Documents of the All-Union Seminar "Current Problems and Mathematical Methods of the Theory of Filtration," Moscow (1984).
8. A. A. Atavin and G. P. Skrebkov, "Simplified method of calculating pressure in a hydraulic system with a compensator," Vestn. Mashinostr., No. 8 (1962).
9. A. D. Al'tshul', Fluid Friction [in Russian], Nedra, Moscow (1982).
10. B. L. Rozhdestvenskii and N. N. Yanenko, Systems of Quasilinear Equations [in Russian], Nauka, Moscow (1978).
11. D. Contractor, "Use of transient regimes of fluid flow in the hydraulic excavation of mineral deposits," Trans. ASME, Teor. Osn. Inzh. Raschetov , No. 2 (1972).
12. A. Kitagawa, T. Takenaka, and Y. Kato, "Study of the high pressure generation by means of oil hammer," Bull. JSME, 27, 234 (1984).

NUMERICAL INVESTIGATION OF THE PROCESS OF SHOCK REFLECTION FROM A WALL WITH A SLOT HOLE

A. B. Britan, A. Ya. Rudnitskii, and A. M. Starik

UDC 533.6.07

The motion of shocks in channels of variable section is an important, hardly studied phenomenon that is utilized extensively in industrial technology, aerophysical experiment practice, and also in laboratory investigations utilizing shock tubes [1].

In the simplest case when two rectilinear channels of differing transverse dimensions are connected by a junction with a smooth change in section, analysis of the flow on both sides of the junction is ordinarily conducted within the framework of the quasistationary one-dimensional stream model [2-4]. In particular, for a channel with diminution of the cross-sectional area A quasistationary theory predicts four possible modifications of the flow wave structure, displayed schematically in the upper part of Fig. 1. Since the flow in the junction itself is not considered, the function is replaced by a discontinuity in the junction diagrams, at which the incident shock arrives from the left (the solid heavy lines are shock trajectories in space–time coordinates). The mode 1 with a reflected and passed shock between which the space is separated by a contact surface (its trajectory is shown by dashed lines in the diagrams) is realized for a subsonic stream velocity. As the incident shock intensity increases a nonstationary rarefaction wave (dash–dot) appears in the stream and accelerates the stream behind the passed wave to a supersonic velocity, mode 2. The reflected shock attenuates for small channel contractions and sufficiently high gas velocities, it ceases to move upstream, mode 3, and degenerates in the long run into a weak disturbance, mode 4 [2, 4]. The flow wave structure in mode 4 is determined by the passed shock and the rarefaction wave.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 112-118, November-December, 1987. Original article submitted July 28, 1986.

